5.D: the ***PhilosophyIdeas*** overview of

Assumptions for Logic

No intellectual system can get started without presuppositions. Logic, even though it aims to capture the foundations of reason, seems to be no exception. Philosophers, however, like to study presuppositions, rather than just accepting them, since they may lead us to an even more fundamental understanding than the topic itself. Hence we have 'meta-logic', which employs its own language to specify and discuss the principles assumed in working logical systems. Four principles are at the centre of metalogical debate: that interpretations of logic can only refer to formulas being either 'true' or 'false'; that every formula in a logic must have one of these two truth-values; that a formula can never simultaneously have both truth-values; and that objects and predicates in a logic can be perfectly identical to one another. These assumptions are typical of 'classical' logic, and rival systems tend to challenge some of them.

The first assumption ('**Bivalence**' – that T and F are the only truth-values) is a semantic principle of meta-logic, saying what is available. The second assumption ('excluded middle' – that for every statement P, we have either P or not-P) is a principle of the actual logic. The appeal of Bivalence is obvious. If someone asserts 'God exists', the ensuing debate seems paralysed if it is not assumed that this must be either True or False. If someone says the sentence is a 'maybe', that says something about them, rather than about the sentence. Classical logic assumes Bivalence, and a very neat system results, in which all of the connectives can be defined by truth tables, containing just Ts and Fs, and the connectives can then all be defined in terms of one another. The disjunction connective ($A \vee B$, where '$\vee$' means 'or') relies for its normal meaning on each of A and B being either T or F, and it is unclear what negation ($\neg A$, where '$\neg$' means 'not') might mean if we can't say that if A is T, then $\neg A$ has to be F (the only other truth-value available).

Bivalence is controversial, because it may imply either over-confidence about what we can assert, or a simplification of reality. The charge of over-confidence is mainly voiced by Intuitionistic logicians, who want restraints placed on what we claim or reason about. The charge arises from the idea that only proof gives security in mathematics, and so in logic there must be some way to establish whether a formula is true or false, and if there is no such way, then reasoning must stop. Thus it is standard in classical logic that $\neg\neg A \rightarrow A$, meaning that if it is not not-raining, then it must be raining. This is because if $\neg\neg A$ is true, then $\neg A$ must be false, and hence A must be true, making two uses of the fact that negation flips the truth-value from F to T or T to F (as they are the only values available). But I may want to say that a place is not not-dangerous, without being able to establish that it is actually (positively) dangerous (if someone was injured there a long time ago). On sliding scales double negation says something different from plain assertion. Other problems are claims about terms that have no reference (like 'unicorns'), or large infinities, or hypothetical claims, or claims about the past. The biggest difficulty for bivalence centres on vague objects or predicates, indicated by our discomfort with aggressive requests for a yes/no answer (to 'am I good-looking?', for example, or 'is that group of people a crowd?'). Insisting on bivalence here seems to distort our picture of reality.

The intuitionist disallows reasoning when T or F are not established, but another strategy is to introduce a third value (such as 'U', for 'undecided'), and proceed with three values (or even more). One might also respond by making bivalence the test of a 'statement' or 'assertion', with any non-bivalent statement being relegated to language about which you cannot reason (such as commands or requests).

There is some confusion about the relation between Bivalence and **Excluded Middle**, mainly over whether the latter is a syntactic or semantic concept (that is, does it concern rules of inference, or truth?). The modern approach is to treat it as syntactic, meaning that P can only be affirmed or denied, so that in any argument we must either accept a proposition or its negation. The metalogical 'or' here is 'exclusive' (meaning one or other, but not both), unlike the normal '$\vee$' connective, which is 'inclusive'. This is because the most basic assumption of logic, **non-contradiction**, prohibits accepting both P and $\neg P$. To reject the assumption that contradictions are forbidden is to accept 'truth-value gluts' (with an overlap of T and F), and this rather surprising proposal is explored in 'paraconsistent' logics.

If we add a semantics to the syntactic Excluded Middle (and there seems little point to a logic with no semantics) then since Excluded Middle says P or $\neg P$, the more cautious semantics says it means P is true or not-true (leaving it open how we understand 'not true'). A bolder semantics says 'not true' must mean 'false', accepting the principle of Bivalence, and only then is P either T or F. Classical logic accepts his bivalent semantics. Note, though, that it is possible to retain excluded middle, while rejecting bivalence.

Standard Excluded Middle assumes that all propositions are either True or False.. For instance, we can probably never know the truth of 'Plato had blue eyes', but standard excluded middle says it is either true or false. Hence realists usually embrace bivalent excluded middle, and anti-realists reject it. Anti-realists say bivalent excluded middle is much too optimistic (and may even need ominiscience!). Also, if you say 'this sentence is not true', that is false if it is true, and true if it is false, so the Liar Paradox is a major problem case for excluded middle, since it can't obey the rule The rejection of excluded middle, though, has big implications. Scientific enquiry seems to assume that there is a determinate true-or-false answer to the questions being asked, and standard logical proof of a proposition relies on the idea that if you show that it can't be false then that means it has to be true. Classical logic more or less collapses if the principle of excluded middle is rejected.

The basic versions of Propositional Logic and Predicate Calculus do not include a symbol (such as '=') saying that two things are **identical**. In ordinary speech we might say that I can meet the identical person today that I met last year, or that you and I drive identical cars, but this is not the strict idea of identity needed for logic and mathematics. In algebra we need to able to say 'x=y', if those variables have identical values. In a logical domain, though, each object has a unique name, so '=' would indicate either a platitude (objects are self-identical) or a falsehood (different things are the same). Nevertheless we normally assume Predicate Calculus 'with identity', in order to deal with the hypothesis that two apparent things are actually one (perhaps under different descriptions).